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When semantics turns to substance: reformulating QCD analysis of $F_2^\gamma(x, Q^2)$

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Abstract: QCD analysis of F_2^γ is revisited. It is emphasized that the presence of the inhomogeneous term in the evolution equations for quark distribution functions of the photon implies important difference in the way factorization mechanism works in photon-hadron and photon-photon collisions as compared to the hadronic ones. Moreover, a careful definitions of the very concepts of the "leading" and "next-to-leading" order QCD analysis of F_2^γ are needed in order to separate genuine QCD effects from those of pure QED origin. Starting from such definitions, I present an alternative approach towards the QCD analysis of F_2^γ that differs in some important aspects from the conventional one. In this approach the NLO QCD analysis of F_2^γ requires the knowledge of two so far uncalculated quantities, and thus is currently impossible to perform. At the LO four known, but in the conventional approach unused quantities are needed. Phenomenological consequences of these differences are discussed.

Keywords: Electromagnetic Processes and Properties, LEP HERA and SLC Physics, Deep Inelastic Scattering, Parton Model.

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1. Introduction

Observed from a large distance photon behaves as a neutral structureless object governed by the laws of Quantum Electrodynamics. However, when probed at short distances it exhibits some properties characteristic of hadrons. This "photon structure" is quantified, as in the case of hadrons, in terms of parton distribution functions (PDF), satisfying certain evolution equations. Because of the direct coupling of photons to quark-antiquark pairs these evolution equations are, contrary to the case of hadrons, inhomogeneous. This inhomogeneity has important implications for QCD analysis of F_2^γ and other physical quantities involving photons in the initial state.

In the course of discussions of this and related subjects concerning conventional QCD analysis of photon structure, I have realized that one likely source of differing opinions[1] concerning the QCD analysis of F_2^γ was the lack of clear and generally accepted definition of what the "leading" and "next-to-leading" approximations mean in the context of photonic interactions. The point is that the conventional QCD analysis of F_2^γ mixes purely QED effects, which are usually quite dominant, with the genuine QCD ones, which represent mostly small corrections only. I therefore start by proposing a definition of these notions for parton distribution functions of the photon and for F_2^γ. In the next step I construct explicit solutions of the inhomogeneous evolution equation including inhomogeneous as well as homogeneous splitting functions up to the order α_s^2. Straightforward analysis of these solutions reveals important differences of the proposed approach with respect to the conventional one: the NLO analysis of F_2^γ is currently impossible to perform because it involves quantities that have not yet been calculated, whereas the LO one requires the inclusion of several additional, but known, $\mathcal{O}(\alpha_s)$ quantities.

The paper is organized as follows: in the next section basic facts and notation concerning PDF of the photon are recalled, followed in section 3 by the discussion of the properties of the pointlike part of quark distribution functions. In section 4 factorization scale dependence of F_2^γ is briefly discussed. In section 5 a reformulation of the "leading" and "next-to-leading" order QCD analysis of F_2^γ is proposed and pointlike solutions of the inhomogeneous evolution equation for non-singlet quark distribution functions of the photon are explicitly written down up to the order α_s^2. Phenomenological implications of the present analysis for F_2^γ are outlined in section 6, followed by the summary and conclusions in section 7.

2. Notation and basic facts

In QCD the coupling of quarks and gluons is characterized by the renormalized colour coupling ("couplant" for short) $\alpha_s(\mu)$, depending on the *renormalization scale* μ and satisfying the equation

$$\frac{\mathrm{d}\alpha_s(\mu)}{\mathrm{d}\ln\mu^2} \equiv \beta\big(\alpha_s(\mu)\big) = -\frac{\beta_0}{4\pi}\,\alpha_s^2(\mu) - \frac{\beta_1}{16\pi^2}\,\alpha_s^3(\mu) + \cdots\,, \tag{2.1}$$

where, in QCD with n_f massless quark flavours, the first two coefficients, $\beta_0 = 11 - 2n_f/3$ and $\beta_1 = 102 - 38n_f/3$, are unique, while all the higher-order ones are ambiguous. As we shall stay in this paper within the NLO QCD, only the first two, *unique*, terms in (2.1) will be taken into account in the following. However, even for a

[1]Compare, for instance, the recent review [1] with [2] and [3].

given r.h.s. of (2.1) its solution $\alpha_s(\mu)$ is not a unique function of μ, because there is an infinite number of solutions of (2.1), differing by the initial condition. This so called *renormalization scheme* (RS) ambiguity[2] can be parameterized in a number of ways. One of them makes use of the fact that in the process of renormalization another dimensional parameter, denoted usually Λ, inevitably appears in the theory. This parameter depends on the RS and at the NLO even fully specifies it. For instance, $\alpha_s(\mu)$ in the familiar MS and $\overline{\mathrm{MS}}$ RS are two solutions of the same equation (2.1), associated with different Λ_{RS}.[3] In this paper we shall work in the standard $\overline{\mathrm{MS}}$ RS of the couplant.

"Dressed" PDF[4] result from the resummation of multiple parton emissions off the corresponding "bare" parton distributions. As a result of this resummation PDF acquire dependence on the *factorization scale* M. This scale defines the upper limit on some measure t of the off-shellness of partons included in the definition of $D(x, M)$

$$D_i(x,M) \equiv \int_{t_{\mathrm{min}}}^{M^2} \mathrm{d}t\, d_i(x,t), \quad i = q, \overline{q}, G\,, \tag{2.2}$$

where the *unintegrated* PDF $d_i(x,t)$ describe distribution functions of partons with the momentum fraction x and *fixed* off-shellness t. Parton virtuality $\tau \equiv |p^2 - m^2|$ or transverse mass $m_T^2 \equiv p_T^2 + m^2$, are two standard choices of such a measure. Because at small t, $d_i(x,t) = \mathcal{O}(1/t^k), k = 1,2$, the dominant part of the integral (2.2) comes from the region of small off-shellness t. Varying the upper bound M^2 in (2.2) has therefore only a small effect on the integral (2.2), leading to weak (at most logarithmic) scaling violations. Factorization scale dependence of PDF of the photon[5] is determined by the system of coupled inhomogeneous evolution equations

$$\frac{\mathrm{d}\Sigma(x,M)}{\mathrm{d}\ln M^2} = \delta_\Sigma k_q + P_{qq} \otimes \Sigma + P_{qG} \otimes G\,, \tag{2.3}$$

$$\frac{\mathrm{d}G(x,M)}{\mathrm{d}\ln M^2} = k_G + P_{Gq} \otimes \Sigma + P_{GG} \otimes G\,, \tag{2.4}$$

$$\frac{\mathrm{d}q_{\mathrm{NS}}(x,M)}{\mathrm{d}\ln M^2} = \delta_{\mathrm{NS}} k_q + P_{\mathrm{NS}} \otimes q_{\mathrm{NS}}\,, \tag{2.5}$$

where[6]

$$\Sigma(x,M) \equiv \sum_{i=1}^{n_f} q_i^+(x,M) \equiv \sum_{i=1}^{n_f} \Big[q_i(x,M) + \overline{q}_i(x,M) \Big], \tag{2.6}$$

[2]In higher orders this ambiguity includes also the arbitrariness of the coefficients β_i, $i \geq 2$.

[3]At the NLO the variation of both the renormalization scale μ and the renormalization scheme $\mathrm{RS} \equiv \{\Lambda_{\mathrm{RS}}\}$ is redundant. It suffices to fix one of them and vary the other, but I stick to the common practice of considering both of them as free parameters.

[4]In the following the adjective "dressed" will be dropped.

[5]If not stated otherwise all distribution functions in the following concern the photon.

[6]For non-singlet quark distribution function q_{NS} another definition is also used in the literature [4]. The definition adopted here corresponds to that introduced in [5].

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$$q_{\rm NS}(x,M) \equiv \sum_{i=1}^{n_f} \left(e_i^2 - \langle e^2\rangle\right)\Big(q_i(x,M) + \overline{q}_i(x,M)\Big), \tag{2.7}$$

$$\delta_{\rm NS} = 6n_f\left(\langle e^4\rangle - \langle e^2\rangle^2\right), \qquad \delta_\Sigma = 6n_f\langle e^2\rangle\,. \tag{2.8}$$

To order α the splitting functions P_{ij} and k_i are given as power expansions in $\alpha_s(M)$

$$k_q(x,M) = \frac{\alpha}{2\pi}\left[k_q^{(0)}(x) + \frac{\alpha_s(M)}{2\pi}k_q^{(1)}(x) + \left(\frac{\alpha_s(M)}{2\pi}\right)^2 k_q^{(2)}(x) + \cdots\right], \tag{2.9}$$

$$k_G(x,M) = \frac{\alpha}{2\pi}\left[\frac{\alpha_s(M)}{2\pi}k_G^{(1)}(x) + \left(\frac{\alpha_s(M)}{2\pi}\right)^2 k_G^{(2)}(x) + \cdots\right], \tag{2.10}$$

$$P_{ij}(x,M) = \frac{\alpha_s(M)}{2\pi}P_{ij}^{(0)}(x) + \left(\frac{\alpha_s(M)}{2\pi}\right)^2 P_{ij}^{(1)}(x) + \cdots, \tag{2.11}$$

where the leading order splitting functions $k_q^{(0)}(x) = (x^2 + (1-x)^2)$ and $P_{ij}^{(0)}(x)$ are *unique*, while all higher-order ones $k_q^{(j)}, k_G^{(j)}, P_{kl}^{(j)}, j \geq 1$ depend on the choice of the *factorization scheme* (FS). Equations (2.3)–(2.5) can be recast into evolution equations for $q_i(x,M)$, $\overline{q}_i(x,M)$ and $G(x,M)$ with inhomegenous splitting functions $k_{q_i}^{(0)} = 3e_i^2 k_q^{(0)}$. The photon structure function $F_2^\gamma(x,Q^2)$, is given as the sum of convolutions

$$\begin{aligned}\frac{1}{x}F_2^\gamma(x,Q^2) = {}& q_{\rm NS}(M)\otimes C_q(Q/M) + \frac{\alpha}{2\pi}\delta_{\rm NS}C_\gamma + \langle e^2\rangle\Sigma(M)\otimes C_q(Q/M) + \\ &+\frac{\alpha}{2\pi}\langle e^2\rangle\delta_\Sigma C_\gamma + \langle e^2\rangle\frac{\alpha_s}{2\pi}G(M)\otimes C_G(Q/M)\end{aligned} \tag{2.12}$$

of photonic PDF and coefficient functions $C_q(x), C_G(x), C_\gamma(x)$ admitting perturbative expansions

$$C_q(x,Q/M) = \delta(1-x) + \frac{\alpha_s(\mu)}{2\pi}C_q^{(1)}(x,Q/M) + \cdots, \tag{2.13}$$

$$C_G(x,Q/M) = \frac{\alpha_s(\mu)}{2\pi}C_G^{(1)}(x,Q/M) + \cdots, \tag{2.14}$$

$$C_\gamma(x,Q/M) = C_\gamma^{(0)}(x,Q/M) + \frac{\alpha_s(\mu)}{2\pi}C_\gamma^{(1)}(x,Q/M) + \cdots, \tag{2.15}$$

where the standard formula for $C_\gamma^{(0)}$ reads

$$C_\gamma^{(0)}(x,Q/M) = \left(x^2 + (1-x)^2\right)\left[\ln\frac{M^2}{Q^2} + \ln\frac{1-x}{x}\right] + 8x(1-x) - 1\,. \tag{2.16}$$

The renormalization scale μ, used as argument of $\alpha_s(\mu)$ in (2.13)–(2.15) is in principle independent of the factorization scale M. Note that despite the presence of μ as argument of $\alpha_s(\mu)$ in (2.13)–(2.15), the coefficient functions C_q, C_G and C_γ are actually independent of μ because the μ-dependence of $\alpha_s(\mu)$ is cancelled by explicit dependence of the coefficient functions $C_q^{(i)}, C_G^{(i)}, C_\gamma^{(i)}, i \geq 2$ on μ [6]. On the other hand, PDF and the coefficient functions C_q, C_G and C_γ do depend on both the factorization scale M and factorization scheme, but in such a correlated manner that physical quantities, like F_2^γ, are independent of both M and the FS, provided expansions (2.9)–(2.11) and (2.13)–(2.15) are taken to all orders in $\alpha_s(M)$ and $\alpha_s(\mu)$. In practical calculations based on truncated forms of (2.9)–(2.11) and (2.13)–(2.15) this invariance is, however, lost and the choice of both M and FS makes numerical difference even for physical quantities. The expressions for $C_q^{(1)}, C_G^{(1)}$ given in [7] are usually claimed to correspond to "$\overline{\mathrm{MS}}$ factorization scheme". As argued in [8], this denomination is, however, incomplete. The adjective "$\overline{\mathrm{MS}}$" concerns exclusively the choice of the RS of the couplant α_s and has nothing to do with the choice of the splitting functions $P_{ij}^{(1)}$. The choices of the renormalization scheme of the couplant α_s and of the factorization scheme of PDF are two independent decisions, concerning two different and in general unrelated redefinition procedures. Both are necessary in order to specify uniquely the results of fixed order perturbative calculations, but we may combine any choice of the RS of the couplant with any choice of the FS of PDF. The coefficient functions C_q, C_G, C_γ depend on both of them, whereas the splitting functions depend only on the latter. The results given in [7] correspond to $\overline{\mathrm{MS}}$ RS of the couplant but to the "minimal subtraction" FS of PDF.[7] It is therefore more appropriate to call this full specification of the renormalization and factorization schemes as "$\overline{\mathrm{MS}}$ + MS scheme". Although the phenomenological relevance of treating μ and M as independent parameters has been demonstrated [10], I follow the usual practice and set $\mu = M$.

3. Pointlike solutions and their properties

The general solution of the evolution eqs. (2.3)–(2.5) can be written as the sum of a particular solution of the full inhomogeneous equation and the general solution of the corresponding homogeneous one, called *hadronic* (HAD) part.[8] A subset of the solutions of full evolution equations resulting from the resummation of series of diagrams in figure 1, which start with the pointlike purely QED vertex $\gamma \to q\overline{q}$, are called *pointlike* (PL) solutions. In writing down the expression for the resummation of diagrams in figure 1 there is a freedom in specifying some sort of boundary condition. It is common to work within a subset of pointlike solutions specified by the value of the scale M_0 at which they vanish. In general, we can thus write ($D = q, \overline{q}, G$)

$$D(x, M) = D^{\mathrm{PL}}(x, M) + D^{\mathrm{HAD}}(x, M)\,. \tag{3.1}$$

Due to the fact that there is an infinite number of pointlike solutions $q^{\mathrm{PL}}(x, M)$, the separation of quark and gluon distribution functions into their pointlike and hadronic

[7]See section 2.6 of [9], in particular eq. (2.31), for discussion of this point.

[8]Sometimes also called "VDM part" because it is usually modelled by PDF of vector mesons.

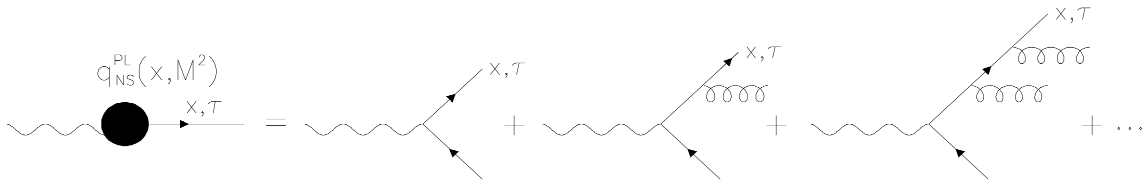

Figure 1: Diagrams defining the pointlike parts of NS quark distribution function in LL approximation. The resummation involves integration over parton virtualities $M_0^2 \le \tau \le M^2$.

parts is ambiguous and therefore these concepts have separately no physical meaning. In [11, 12] we discussed numerical aspects of this ambiguity for the Schuler-Sjöstrand sets of parameterizations [13] of PDF of real as well as virtual photons. To see the most important feature of the pointlike part of quark distribution functions that will be crucial for the following analysis, let us consider in detail the pointlike part of the non-singlet quark distribution function $q_{\mathrm{NS}}(x, M)$, which is explicitly defined via the series

$$\begin{aligned} q_{\mathrm{NS}}^{\mathrm{PL}}(x, M_0, M) \equiv & \\ \equiv \frac{\alpha}{2\pi} k_{\mathrm{NS}}^{(0)}(x) \int_{M_0^2}^{M^2} \frac{\mathrm{d}\tau}{\tau} + \int_x^1 \frac{\mathrm{d}y}{y} P_{qq}^{(0)}\left(\frac{x}{y}\right) \int_{M_0^2}^{M^2} \frac{\mathrm{d}\tau_1}{\tau_1} \frac{\alpha_s(\tau_1)}{2\pi} \frac{\alpha}{2\pi} k_{\mathrm{NS}}^{(0)}(y) \int_{M_0^2}^{\tau_1} \frac{\mathrm{d}\tau_2}{\tau_2} + & \\ + \int_x^1 \frac{\mathrm{d}y}{y} P_{qq}^{(0)}\left(\frac{x}{y}\right) \int_y^1 \frac{\mathrm{d}w}{w} P_{qq}^{(0)}\left(\frac{y}{w}\right) \int_{M_0^2}^{M^2} \frac{\mathrm{d}\tau_1}{\tau_1} \frac{\alpha_s(\tau_1)}{2\pi} \times & \\ \times \int_{M_0^2}^{\tau_1} \frac{\mathrm{d}\tau_2}{\tau_2} \frac{\alpha_s(\tau_2)}{2\pi} \frac{\alpha}{2\pi} k_{\mathrm{NS}}^{(0)}(w) \int_{M_0^2}^{\tau_2} \frac{\mathrm{d}\tau_3}{\tau_3} + \cdots , & \end{aligned} \tag{3.2}$$

where $k_{\mathrm{NS}}^{(0)}(x) = \delta_{\mathrm{NS}} k_q^{(0)}(x)$. In terms of moments defined as

$$f(n) \equiv \int_0^1 x^n f(x)\, \mathrm{d}x \tag{3.3}$$

this series can be resummed in a closed form

$$q_{\mathrm{NS}}^{\mathrm{PL}}(n, M_0, M) = \frac{4\pi}{\alpha_s(M)} \left[1 - \left(\frac{\alpha_s(M)}{\alpha_s(M_0)} \right)^{1 - 2P_{qq}^{(0)}(n)/\beta_0} \right] a_{\mathrm{NS}}(n) , \tag{3.4}$$

where

$$a_{\mathrm{NS}}(n) \equiv \frac{\alpha}{2\pi\beta_0} \frac{k_{\mathrm{NS}}^{(0)}(n)}{1 - 2P_{qq}^{(0)}(n)/\beta_0} . \tag{3.5}$$

It is straightforward to show that (3.2) or, equivalently, (3.4), satisfy the evolution eq. (2.5) with the splitting functions k_q and P_{ij} including the first terms $k_q^{(0)}$ and $P_{qq}^{(0)}$ only.

Transforming (3.5) to the x-space by means of inverse Mellin transformation we get $a_{\rm NS}(x)$ shown in figure 2. The resummation softens the x-dependence of $a_{\rm NS}(x)$ with respect to the first term in (3.2), proportional to $k_{\rm NS}^{(0)}(x)$, but does not change the logarithmic dependence of $q_{\rm NS}$ on M. In the non-singlet channel the effects of gluon radiation on $q_{\rm NS}^{\rm PL}$ are significant for $x \gtrsim 0.6$ but small elsewhere, whereas in the singlet channel such effects are marked also for $x \lesssim 0.5$, where they lead to a steep rise of $xq_{\Sigma}^{\rm PL}(x,M)$ at very small x. As emphasized long time ago by the authors of [2] the logarithmic dependence of $q_{\rm NS}^{\rm PL}$ and $q_{\Sigma}^{\rm PL}$ on M has nothing to do with QCD and results from integration over the transverse momenta (virtualities) of quarks coming from the basic QED $\gamma \to q\overline{q}$ splitting. For $M/M_0 \to \infty$ the second term in brackets of (3.4) vanishes and therefore all pointlike solutions share the same large M behaviour

$$q_{\rm NS}^{\rm PL}(x,M_0,M) \longrightarrow \frac{4\pi}{\alpha_s(M)} a_{\rm NS}(x) \equiv q_{\rm NS}^{\rm AP}(x,M) \propto \ln\frac{M^2}{\Lambda^2}\,, \tag{3.6}$$

defining the so-called *asymptotic pointlike* solution [14, 15]. Note that (3.6) can be interpreted as a special case of the general pointlike solution (3.4) in which the lower integration limit M_0 in (3.2) is set equal to Λ, i.e. $M_0 = \Lambda$! The fact that for the asymptotic pointlike solution (3.6) $\alpha_s(M)$ appears in the denominator of (3.6) has often been interpreted as evidence that $q(x,M) = \mathcal{O}(\alpha/\alpha_s)$. This interpretation cannot, however, be taken literally. It is obvious that provided M_0 is kept fixed

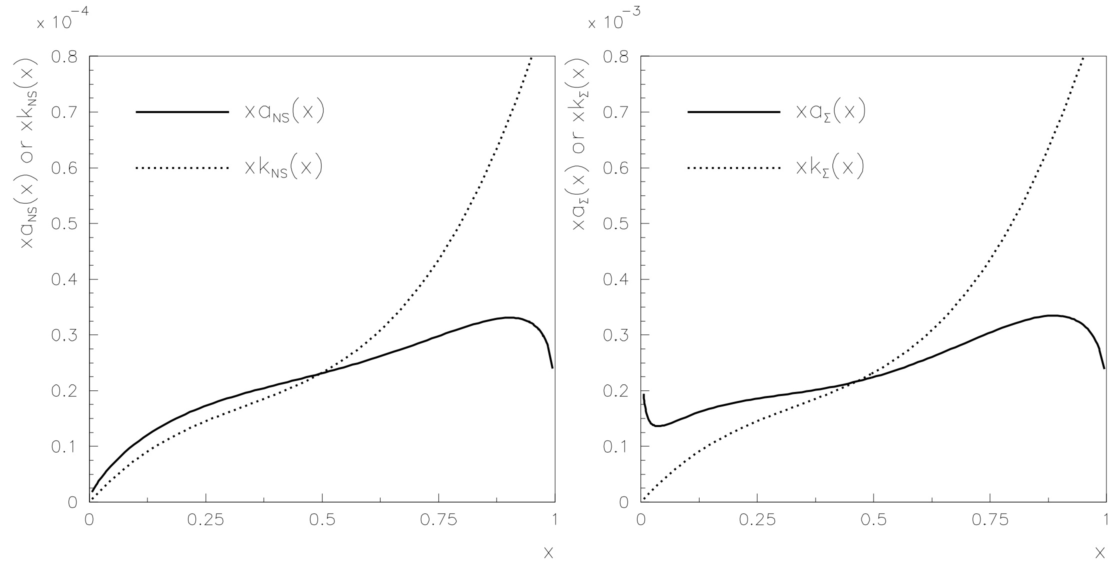

Figure 2: Comparison, left in the non-singlet and right in the singlet channels, of the functions $xk_i(x) = x\delta_i k_q^{(0)}$, $i = \mathrm{NS}, \Sigma$ with the functions $xa(x)$ corresponding to asymptotic pointlike solutions (3.6) and its analogue in the singlet channel.

when $\alpha_s \to 0$ the sum (3.2) approaches its first term [16]

$$q_{\rm NS}^{\rm PL}(x,M,M_0) \longrightarrow \frac{\alpha}{2\pi}k_{\rm NS}^{(0)}(x)\ln\frac{M^2}{M_0^2}\,, \tag{3.7}$$

corresponding to purely QED splitting $\gamma \to q\overline{q}$. However, the claim that $q \propto \alpha/\alpha_s$ is questionable even for the asymptotic pointlike solution (3.6). The fact that the latter diverges for $\Lambda \to 0$ is a direct consequence of the fact that for this (and only this) pointlike solution the decrease of the coupling $\alpha_s(M/\Lambda)$ as $\Lambda \to 0$ is compensated by the simultaneous extension of the integration region as $M_0 = \Lambda \to 0$! If, however, QCD is switched off by sending $\Lambda \to 0$ *without* extending the integration region, i.e. for *fixed* M_0, there is no trace of QCD left and we get back the simple QED formula (3.7). This suggests to identify QCD contribution to $q_{\rm NS}^{\rm PL}$ with the difference

$$q_{\rm QCD}(x,M) \equiv q_{\rm NS}^{\rm PL}(x,M_0,M) - \frac{\alpha}{2\pi}k_{\rm NS}^{(0)}(x)\ln\frac{M^2}{M_0^2}\,. \tag{3.8}$$

Expanding (3.4) in powers of $\alpha_s(M)$ keeping M_0 and M fixed we find

$$q_{\rm QCD}(n,M) = \frac{\alpha}{2\pi}\frac{\alpha_s}{2\pi}\frac{k_q^{(0)}(n)P_{qq}^{(0)}(n)}{2}\ln^2\frac{M^2}{M_0^2} + \mathcal{O}(\alpha_s^2)\,, \tag{3.9}$$

as expected from the explicit expression for the second term on the r.h.s. of (3.2).

In summary, $q_{\rm NS}^{\rm PL}(x,M)$ can be written a sum of two terms: purely QED contribution, proportional to α, and the term describing genuine QCD effects, proportional to $\alpha\alpha_s$ and thus vanishing when QCD is switched off. One can write $q \propto \alpha/\alpha_s$ merely as a shorthand for the specification of large M behaviour of (3.2), as expressed in (3.6). In fact, this is what one finds in the original papers [14, 15] which do not contain any explicit claim[9] that $q(x,M) = \mathcal{O}(\alpha/\alpha_s)$.

4. Factorization scale dependence of F_2^γ

In my view the main reason for adhering to this claim is related to the assumption that at the leading order of QCD the observable $F_2^\gamma(x,Q^2)$ is related to $q(x,M)$ in the same way as for hadrons, i.e.

$$F_{2,\rm LO}^\gamma(x,Q^2) = q_{\rm LO}(x,M)\,. \tag{4.1}$$

To evaluate the r.h.s. of (4.1) the factorization scale M has to be chosen. The requirement of factorization scale invariance of a physical quantity P implies that the difference $P^{(n)}(M_1) - P^{(n)}(M_2)$ of n-th order perturbative approximants behaves

[9] See, for instance, eq. (3.20) of [15].

as $\mathcal{O}(\alpha_s^{n+1})$. For hadrons (4.1) is consistent with this requirement because for hadrons $q(x,M)$ satisfies homogeneous evolution equation and, consequently, the difference

$$\Delta(M_1,M_2) \equiv q(M_1) - q(M_2) \propto \alpha_s(M_1)\ln\left(\frac{M_1^2}{M_2^2}\right) q(M_1) \tag{4.2}$$

is manifestly of one order of α_s higher that q itself and therefore (4.1) to the order considered independent of the choice of M!

For the photon, however, the presence of the inhomogeneous term $(\alpha/2\pi)k_q^{(0)}$ in evolution equations implies $\Delta(M_1,M_2) \propto \alpha\ln(M_1^2/M_2^2)$ and therefore in order to retain factorization scale invariance of (4.1) the property $q \propto \alpha/\alpha_s$ seems indispensable. The assignment in the conventional approach of $C_\gamma^{(0)}$ to the NLO is justified also by the fact that for fixed ratio $d \equiv M_1/M_2$, the difference $\Delta(M_1,M_2) \propto \alpha\ln d$ is subleading in M_1^2 with respect to $q_{\mathrm{LO}}(M_1) \propto \alpha\ln M_1^2$. This is true but it must be kept in mind that in the conventional approach the dominant $\ln M^2$ behaviour of $q_{\mathrm{LO}}(M)$ comes from QED rather than QCD effects.

There is, however, a different way of guaranteeing the renormalization scale invariance of $F_{2,\mathrm{LO}}^\gamma$ that does not rely on the assumption $q \propto \alpha/\alpha_s$, but modifies the relation (4.1) instead. I will show that this modification is consistent with the fact that $q \propto \alpha$ and demonstrate how the additional terms in this relation conspire to guarantee the factorization scale invariance of $F_{2,\mathrm{LO}}^\gamma$.

5. QCD analysis of F_2^γ

In this section QCD analysis of F_2^γ that separates genuine QCD effects from those of purely QED origin will be presented. Throughout this and following sections I will furthermore restrict my attention to the non-singlet part of F_2^γ in (2.12)

$$\frac{1}{x}F_{2,\mathrm{NS}}^\gamma(x,Q^2) = q_{\mathrm{NS}}(M)\otimes C_q(Q/M) + \frac{\alpha}{2\pi}\delta_{\mathrm{NS}}C_\gamma \tag{5.1}$$

and non-singlet quark distribution function as defined in (2.7). I will consider the contribution of light quarks (i.e. u, d, s) only and, moreover, disregard the difference[10] between their distributions functions after division by e_q^2. Under these simplifying assumptions we have

$$\begin{aligned}\frac{1}{x}F_{2,\mathrm{NS}}^\gamma(x,Q^2) &= \delta_{\mathrm{NS}}\left[q(M)\otimes C_q(Q/M) + \frac{\alpha}{2\pi}C_\gamma(Q/M)\right]\\ &= \delta_{\mathrm{NS}}\left[q(M) + \frac{\alpha_s}{2\pi}q(M)\otimes C_q^{(1)}(Q/M) + \frac{\alpha}{2\pi}C_\gamma^{(0)}(Q/M) + \right.\\ &\qquad\left. + \frac{\alpha}{2\pi}\frac{\alpha_s}{2\pi}C_\gamma^{(1)}(Q/M) + \cdots\right],\end{aligned} \tag{5.2}$$

[10]This difference, which comes entirely from the hadronic part of quark distribution functions, is negligible above roughly $x = 0.25$ but becomes sizable below this value.

where $q(x,M) \equiv u(x,M)/3e_u^2 \doteq d(x,M)/3e_d^2 \doteq s(x,M)/3e_s^2$. For simplicity I will drop the overall charge factor $\delta_{\rm NS}$ as well as the subscript "NS", introduce $F(x,Q^2) \equiv (1/x)F_{2,{\rm NS}}^{\gamma}(x,Q^2)$ and use the dot to denote derivatives with respect to $\ln M^2$ or $\ln Q^2$. In (5.1)–(5.2) I have written out explicitly the symbol $\otimes$ to denote convolutions in x-space, but henceforth I will work mostly in the momentum space where such convolutions become simple multiplications.[11]

5.1 Defining the leading and next-to-leading orders for $F(Q^2)$

For the purpose of this discussion it is useful to recall the meaning of the terms "leading" and "next-to-leading" order of perturbative QCD in the case of the familiar ratio

$$R_{\rm e^+e^-}(Q) \equiv \frac{\sigma({\rm e^+e^-} \to {\rm hadrons})}{\sigma({\rm e^+e^-} \to \mu^+\mu^-)} = \left(3\sum_{i=1}^{n_f} e_i^2\right)\left(1+r(Q)\right). \tag{5.3}$$

The prefactor

$$R_{\rm QED} \equiv \left(3\sum_{i=1}^{n_f} e_i^2\right) \tag{5.4}$$

comes purely from QED, whereas genuine QCD effects give $r(Q)$ as expansion in powers of α_s

$$r(Q) = \frac{\alpha_s(M)}{\pi}\left[1+\frac{\alpha_s(M)}{2\pi}r_1(Q/M)+\cdots\right]. \tag{5.5}$$

For the quantity (5.3) it is a generally accepted procedure to divide out the QED contribution $R_{\rm QED}$ and apply the terms "leading" and "next-to-leading" orders to QCD analysis of $r(Q)$. I will now present the organization of QCD expression for $F(Q^2)$ which follows closely the procedure used in QCD analysis of (5.3). Following the discussion at the end of section 3, let us write the pointlike part of $q(M)$, satisfying the condition $q(M_0)=0$, as the sum of two terms

$$q^{\rm PL}(M) = q_{\rm QED}(M) + q_{\rm QCD}(M)\,, \tag{5.6}$$

where the purely QED contribution is given as

$$q_{\rm QED}(M) \equiv \frac{\alpha}{2\pi}k_q^{(0)}\ln\frac{M^2}{M_0^2} \tag{5.7}$$

and M_0, a free parameter specifying the pointlike solution, can be interpreted as the lower limit on the integral over quark virtuality included in the definition of $q^{\rm PL}(M)$.

[11] To save space, their dependence on x or the momentum variable n will not be written out explicitly.

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Defined in this way $q_{\rm QCD}$ is due entirely to QCD effects, i.e. vanishes when we switch off QCD. The expression (5.7) is a close analogue of the QED contribution $R_{\rm QED}$ in (5.4). Note that $q_{\rm QCD}$ satisfies the inhomogeneous evolution equation

$$\dot{q}_{\rm QCD} = \frac{\alpha_s}{2\pi}\left[\frac{\alpha}{2\pi}k_q^{(1)} + P_{qq}^{(0)} q_{\rm QED}\right] + \\ + \left[\left(\frac{\alpha_s}{2\pi}\right)^2 \left(\frac{\alpha}{2\pi}k_q^{(2)} + P_{qq}^{(1)} q_{\rm QED}\right) + \frac{\alpha_s}{2\pi}P_{qq}^{(0)} q_{\rm QCD}\right] + \cdots , \tag{5.8}$$

where the first and second terms are of the order $\alpha\alpha_s$ and $\alpha\alpha_s^2$, respectively. Equation (5.8) differs from that satisfied by the full quark distribution function $q^{\rm PL}$ not only by the absence of the term $(\alpha/2\pi)k_q^{(0)}$ but also by shifted appearance of higher-order coefficients $k_q^{(i)}; i \geq 1$. For instance, the inhomogeneous splitting function $k_q^{(1)}$ enters the evolution equation for $q_{\rm QCD}$ at the same order as homogeneous splitting function $P_{qq}^{(0)}$ and thus these splitting functions will appear at the same order also in its solutions. The fact that $k_q^{(1)}$ is a function of x (or n) only, whereas $q_{\rm QED}$ is also a function of M, influences relative importance of the two terms in the coefficient by α_s in (5.8) as M varies, but does not change the fact that both of them contribute at the same order. The presence of $k_q^{(2)}$ in the $\mathcal{O}(\alpha\alpha_s^2)$ term on the r.h.s. of (5.8) implies that in the proposed approach the NLO QCD analysis of $F(Q^2)$ requires the knowledge of a quantity that has not yet been calculated.

5.1.1 QED part

The purely QED part of $F(Q^2)$ is given by the sum

$$F_{\rm QED}(Q^2) = q_{\rm QED}(M) + \frac{\alpha}{2\pi}C_\gamma^{(0)}(Q/M) . \tag{5.9}$$

It is worth emphasizing that although both terms on the r.h.s. of (5.9) depend on M, their sum is independent of it and is a function of Q^2 and M_0^2 only![12]

Already at this point I start to depart from the conventional approach which assigns $C_\gamma^{(0)}$ to the NLO QCD corrections. However, as $C_\gamma^{(0)}$ has nothing to do with QCD and is due solely to QED effects, it should be part of any QCD analysis of experimental data. I therefore disagree with the claim in [17], that the SaSM parameterizations are "theoretically inconsistent" because they combine the NLO quantity $C_\gamma^{(0)}$ with the LO quark distribution functions. From the point of view of including $C_\gamma^{(0)}$ in analyses of $F(x,Q^2)$, the SaSM parameterizations are closer to complete LO QCD formula in my approach than analogous analyses using SaSD sets.

Let me reiterate that the principal feature of $F(Q^2)$, namely its logarithmic rise with Q^2, is basically a QED effect and consequently its observation in experiments by itself no verification of QCD. One has to go to subtler features, like the deviation of the x-dependence of $\mathrm{d}F(x,Q^2)/\mathrm{d}\ln Q^2$ from the QED result $(\alpha/2\pi)k_q^{(0)}(x)$, to identify genuine QCD effects!

[12]The dependence on M_0 must not be confused with factorization scale dependence.

5.1.2 QCD part

Following the QCD analyses of (5.3) $F(Q^2)$ can be rewritten as the sum of its QED and QCD parts

$$\begin{aligned} F(Q^2) = & \underbrace{q_{\rm QED} + \frac{\alpha}{2\pi}C_\gamma^{(0)}}_{\rm A_0;\ pure\ QED} + \\ & + \underbrace{q_{\rm QCD} + \frac{\alpha_s}{2\pi}C_q^{(1)}q_{\rm QED} + \frac{\alpha}{2\pi}\frac{\alpha_s}{2\pi}C_\gamma^{(1)}}_{\equiv A_1,\ \text{starting as } \mathcal{O}(\alpha\alpha_s)} + \\ & + \underbrace{\frac{\alpha_s}{2\pi}C_q^{(1)}q_{\rm QCD} + \frac{\alpha}{2\pi}\left(\frac{\alpha_s}{2\pi}\right)^2 C_\gamma^{(2)} + \left(\frac{\alpha_s}{2\pi}\right)^2 C_q^{(2)}q_{\rm QED}}_{\equiv A_2,\ \text{starting as } \mathcal{O}(\alpha\alpha_s^2)} + \cdots \end{aligned} \tag{5.10}$$

In the above expressions I have grouped into quantities A_0, A_1 and A_2 the contributions that start at zero, first and second order of α_s, assuming $q_{\rm QCD} \approx \alpha_s$. In the next subsection I will present explicit solutions demonstrating this behaviour.

The first line in (5.10) contains the purely QED contribution (5.9). The LO QCD correction to $F(Q^2)$ is identified with A_1,

$$F_{\rm LO}(Q^2) = q_{\rm QCD} + \frac{\alpha_s}{2\pi}C_q^{(1)}q_{\rm QED} + \frac{\alpha}{2\pi}\frac{\alpha_s}{2\pi}C_\gamma^{(1)} \tag{5.11}$$

and the NLO one with the sum $A_1 + A_2$. Beside its more complicated structure, eq. (5.11) differs from $r_{\rm LO} = \alpha_s/\pi$, the LO contribution to (5.5), by the fact that whereas the latter is unique, (5.11) depends on M_0. Any QCD analysis of $F(Q^2)$ must therefore start with fixing the value of this parameter. For heavy quarks M_0^2 can be related to m_Q^2 and for virtual photons to their virtuality P^2.

Returning to the last paragraph of section 4, let us now verify that the factorization scale invariance does, indeed, hold for the expression (5.11) with $q_{\rm QCD}$ satisfying (5.8) up to the order $\alpha\alpha_s$. Taking the derivative of (5.11) with respect to $\ln M^2$ we find, taking into account that from similar considerations in hadronic collisions we know that $\dot{C}_q^{(1)} = -P_{qq}^{(0)}$,

$$\dot{F}_{\rm LO} = \frac{\alpha}{2\pi}\frac{\alpha_s}{2\pi}\left[k_q^{(1)} + C_q^{(1)} + \dot{C}_\gamma^{(1)}\right] + \mathcal{O}(\alpha\alpha_s^2)\,. \tag{5.12}$$

Contrary to the case of hadronic structure functions $F^{\rm HAD}$, where analogous derivative is proportional to $\alpha_s F^{\rm HAD}$ and thus manifestly of the NLO, for $F_{\rm LO}(Q^2)$ the condition that (5.12) is of the order $\alpha\alpha_s^2$ implies the following relation:

$$k_q^{(1)} + C_q^{(1)} + \dot{C}_\gamma^{(1)} = 0\,. \tag{5.13}$$

Because $C_\gamma^{(1)}$ depends on $\ln M^2$ as $\ln^2(Q^2/M^2)$ (5.13) actually implies two non-trivial relations. Their validity can be verified using the expressions for the coefficients $C_q^{(1)}$

and $C_G^{(1)}$, calculated in [18, 19]. According to [20] and taking into account different normalization conventions,[13] $C_\gamma^{(1)}$ can be obtained from $C_G^{(1)}$ in [19] by

- replacing $T_f n_f$ by unity,
- dropping terms including $P_{GG}^{(0)}$ and β_0, which are absent for $C_\gamma^{(1)}$, and
- replacing $P_{qG}^{(0)}$ with $k_q^{(1)}$.

Substituting $C_\gamma^{(1)}$ obtained in this way into (5.13) one finds that the sum on its l.h.s. does, indeed, vanish. The presence of $k_q^{(1)}$ in (5.13) is vital for this cancellation.

For the purpose of comparing (5.11) with the conventional LO formula for $F(Q^2)$ as well as with the data, it is necessary to add to it the QED contribution (5.9)

$$F_{\mathrm{QED}} + F_{\mathrm{LO}} = q_{\mathrm{QED}} + q_{\mathrm{QCD}} + \frac{\alpha}{2\pi} C_\gamma^{(0)} + \frac{\alpha_s}{2\pi} C_q^{(1)} q_{\mathrm{QED}} + \frac{\alpha}{2\pi}\frac{\alpha_s}{2\pi} C_\gamma^{(1)} . \tag{5.14}$$

Recalling the conventional formulation (4.1) of F_{LO}, which includes the QED part and in which $q_{\mathrm{LO}}(M)$ satisfies non-singlet evolution eq. (2.5) with $k_q^{(0)}$ and $P_{qq}^{(0)}$ splitting functions only, we see that it differs from (5.14)

- by the absence of the contributions of photonic coefficient functions $C_\gamma^{(0)}$ and $C_\gamma^{(1)}$,
- by the absence of the convolution of quark coefficient function $C_q^{(1)}$ with q_{QED}, and
- by the fact $k_q^{(1)}$ is not included in the evolution equation for $q(M)$.

These differences are quite important, but as all the mentioned additional quantities are known, there is no obstacle to performing LO QCD analysis in the proposed approach using (5.14) with q_{QCD} given by the formula (5.29) of subsection 5.2. On the other hand, as neither $k_q^{(2)}$ nor $C_\gamma^{(2)}$ are known, the NLO QCD analysis of $F(Q^2)$ is currently impossible to perform in this approach. Note that within the conventional approach the terms in (5.11) proportional to $C_\gamma^{(0)}$ and $C_q^{(1)}$ are part of the NLO expression

$$\frac{1}{x} F_{2,\mathrm{NLO}}^\gamma = q + \frac{\alpha_s}{2\pi} C_q^{(1)} q + \frac{\alpha}{2\pi} C_\gamma^{(0)} , \tag{5.15}$$

but the photonic coefficient function $C_\gamma^{(1)}$, though known, is not used even at the NLO. As we shall see in section 6, its contribution is, however, numerically comparable to that of $C_\gamma^{(0)}$!

[13] My $P_{ij}^{(0)}$ are by a factor of 4 smaller than those used in [18, 19], and P_{ij} are expanded in powers of $\alpha/2\pi$ in this paper and in powers of $\alpha_s/4\pi$ in [18, 19, 20].

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5.2 Explicit form of the pointlike solution $q^{\rm PL}(n,M,M_0)$

The inhomogeneous evolution equation for the non-singlet quark distribution function $q^{\rm PL}$ is particularly simple to solve in the case $\beta_i = 0, i \geq 1$. As none of the conclusions of this paper depends in any essential way on non-zero value of β_1, and higher-order coefficients β_i; $i \geq 2$ can be set to zero by the choice of an appropriate renormalization scheme, I will set $\beta_i = 0; i \geq 1$ through the rest of this paper. Under this simplifying assumption the derivative with respect to $\ln M^2$ can be replaced with the derivative with respect to $y \equiv \alpha_s(M)$ and the inhomegeneous evolution equation rewritten as

$$\begin{aligned}
q'(y) \equiv \frac{{\rm d}q(y)}{{\rm d}y} &= -\frac{4\pi}{\beta_0}\frac{1}{\alpha_s^2}\frac{{\rm d}q(M)}{{\rm d}\ln M^2} \\
&= -\left[\frac{2\alpha k_q^{(0)}}{\beta_0}\frac{1}{y^2} + \frac{\alpha k_q^{(1)}}{\pi\beta_0}\frac{1}{y} + \frac{\alpha k^{(2)}}{2\pi^2\beta_0} + \cdots\right] - \\
&\quad - \left[\frac{2P_{qq}^{(0)}}{\beta_0}\frac{1}{y} + \frac{P_{qq}^{(1)}}{\pi\beta_0} + \frac{P_{qq}^{(2)}}{2\pi^2\beta_0}y + \cdots\right] q\,, \\
&= -Q(y) - P(y)q(y)\,,
\end{aligned} \tag{5.16}$$

where

$$P(y) = \frac{2P_{qq}^{(0)}}{\beta_0}\frac{1}{y} + \frac{P_{qq}^{(1)}}{\pi\beta_0} + \frac{P_{qq}^{(2)}}{2\pi^2\beta_0}y + \cdots = \sum_{i=1}^{\infty} p_i\, y^{i-2}\,, \tag{5.17}$$

$$Q(y) = \frac{2\alpha k^{(0)}}{\beta_0}\frac{1}{y^2} + \frac{\alpha k^{(1)}}{\pi\beta_0}\frac{1}{y} + \frac{\alpha k^{(2)}}{2\pi^2\beta_0} + \cdots = \sum_{i=1}^{\infty} q_i\, y^{i-3}\,. \tag{5.18}$$

This standard inhomogeneous differential equation of the type

$$q'(y) + P(y)q(y) + Q(y) = 0 \tag{5.19}$$

has a general solution in the form

$$q(y,y_0) = \exp\left(-\int_{y_0}^{y} P(u){\rm d}u\right)\left[q_0 - \int_{y_0}^{y} Q(v)\exp\left(\int_{y_0}^{v} P(u){\rm d}u\right)\right], \tag{5.20}$$

where $q_0 = q(y_0,y_0) = q(M_0)$ specifies the initial condition imposed on (5.20). Our pointlike solutions correspond to $q_0 = 0$ and we thus have

$$\begin{aligned}
q(y,y_0) &= -\exp\left(-\int_{y_0}^{y} P(u)\,{\rm d}u\right)\int_{y_0}^{y} Q(v)\exp\left(\int_{y_0}^{v} P(u)\,{\rm d}u\right) \\
&= -\int_{y_0}^{y} Q(v)\exp\left(\int_{y}^{v} P(u)\,{\rm d}u\right).
\end{aligned} \tag{5.21}$$

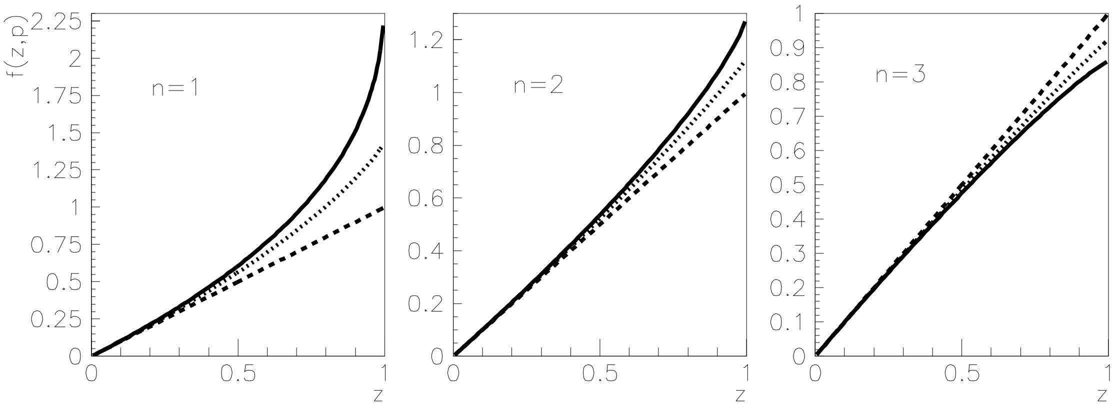

Figure 3: The functions $V(z,p)$ (solid) and $W(z,p)$ (dotted), where $p = -2P_{qq}^{(0)}(n)/\beta_0$, evaluated for $n = 1, 3, 6$ and compared to $f(z) = z$ (dashed lines).

For further discussion it is useful to introduce the following functions:

$$V(z,p) \equiv \frac{1-(1-z)^p}{p} = \frac{\left(1-(y/y_0)^p\right)}{p}\,, \tag{5.22}$$

$$W(z,p) \equiv \frac{-2}{p(1+p)z}\left[1-(1-z)^{1+p}-(1+p)z\right], \tag{5.23}$$

where

$$z \equiv y\left(\frac{1}{y}-\frac{1}{y_0}\right) = yA = \alpha_s(M)\frac{\beta_0}{4\pi}L_M = \frac{\ln(M^2/M_0^2)}{\ln(M^2/\Lambda^2)}\,; \quad L_M \equiv \ln\frac{M^2}{M_0^2}\,, \tag{5.24}$$

which both behave for $z \to 0$ and all p like z. Their shapes are shown, for three values of p corresponding to $n = 1, 3, 6$, in figure 3. For fixed M^2 and M_0^2, $z \to 0$ corresponds $y = \alpha_s \to 0$. Inserting the expansions (5.17) and (5.18) into (5.21) and retaining the first three terms we get

$$\begin{aligned} q(y,y_0) = {} & e^{-\left(p_2 y + p_3 y^2/2\right)} \times \\ & \times \left[q_1 V(z,1-p_1)\frac{1}{y} + q_1\, p_2 V(z,-p_1) + \frac{q_1(p_2^2+p_3)}{2}V(z,-1-p_1)y + \right. \\ & \quad + q_2 V(z,-p_1) + q_2 p_2 V(z,-1-p_1)\,y + \frac{q_2(p_2^2+p_3)}{2}V(z,-2-p_1)\,y^2 + \\ & \quad + q_3 V(z,-1-p_1)\,y + q_3\, p_2 V(z,-2-p_1)\,y^2 + \\ & \quad \left. + \frac{q_3(p_2^2+p_3)}{2}V(z,-3-p_1)\,y^3\right]. \end{aligned} \tag{5.25}$$

Expanding the exponential prefactor in (5.25), adding and subtracting the QED contribution (5.7) and grouping the terms that stand by same power of y we get, keeping terms up to y^2,

$$q(y,y_0)=\frac{\alpha}{2\pi}k_q^{(0)}\ln\frac{M^2}{M_0^2}+ \tag{5.26}$$

$$+\left[\frac{q_1}{y}V(z,1-p_1)-\frac{\alpha}{2\pi}k^{(0)}\ln\frac{M^2}{M_0^2}+q_2V(z,-p_1)\right]+ \tag{5.27}$$

$$+\left[q_1\,p_2\frac{V(z,-p_1)-V(z,1-p_1)}{y}+q_3V(z,-1-p_1)-\right.$$
$$\left.-\,q_2\,p_2V(z,-p_1)\right]y\,. \tag{5.28}$$

The first line in the above expression defines the purely QED part (5.7), the second line the LO and the sum of (5.27) and (5.28) the NLO approximations $q_{\rm LO}$ and $q_{\rm NLO}$. Note that the difference $(V(-p_1)-V(1-p_1))/y$ behaves at small y as $A^2y/2=O(y)$, i.e. in the same way as $V(z,p)\approx z=O(y)$ and therefore contributes to $q_{\rm NLO}$.

5.2.1 QCD part: leading order

In terms of functions $V(z,p)$ and $W(z,p)$ the LO expression for $q_{\rm QCD}$ reads

$$q_{\rm LO}(M)=\frac{\alpha}{2\pi}\frac{k_q^{(0)}P_{qq}^{(0)}}{\beta_0}\ln\frac{M^2}{M_0^2}W(z,-p_1)+\frac{\alpha}{2\pi}\frac{2k_q^{(1)}}{\beta_0}V(z,-p_1)\,. \tag{5.29}$$

The first term in (5.29) coincides with the LO expression (3.4) of the conventional approach from which the purely QED part (5.7) was subtracted. The numerical importance of the second term in (5.29), which is absent in the conventional approach, will be discussed in the next section. In the limit $y=\alpha_s(M/\Lambda)\rightarrow 0$, realized by sending $\Lambda\rightarrow 0$, and keeping Q^2, M^2 and M_0^2 fixed, both terms in (5.29) behave in the same way and we find

$$q_{\rm LO}(M)\approx\left[\frac{q_1p_1A^2}{2}+q_2A\right]\alpha_s(M)=\frac{\alpha}{2\pi}\frac{\alpha_s(M)}{2\pi}\left[\frac{k_q^{(0)}P_{qq}^{(0)}}{2}L_M^2+k_q^{(1)}L_M\right]. \tag{5.30}$$

It is easy to check that (5.30) satisfies (5.8) with only the first term on its r.h.s.

The behaviour of $q_{\rm LO}(M)$ at large scales M is determined by (5.29) in the limit $z\rightarrow 1$. Taking into account that $V(1,p)=1/p$ and $W(1,p)=2/(1+p)$ we find

$$q_{\rm LO}(M)\longrightarrow\frac{\alpha}{2\pi}k_q^{(0)}\frac{2P_{qq}^{(0)}}{\beta_0}\frac{1}{1-2P_{qq}^{(0)}/\beta_0}L_M+\frac{\alpha}{2\pi}\frac{k_q^{(1)}}{-P_{qq}^{(0)}}\,. \tag{5.31}$$

The first term in (5.31) is dominant as $M\rightarrow\infty$, but as we shall see in section 6 the second one, absent in conventional LO analysis, is numerically more important in most of the currently accessible kinematical region.

5.2.2 QCD part: next-to-leading order

The expression for $q_{\rm NLO}(M)$, given by the sum of (5.27) and (5.28), behaves at small $y = \alpha_s$ as

$$\begin{aligned} q_{\rm NLO}(M) &\approx \left[\frac{q_1 p_1 A^2}{2} + q_2 A\right]\alpha_s(M) + \left[\frac{q_1 p_2 A^2}{2} + (q_3 - q_2 p_2)A\right]\alpha_s(M)^2 \\ &= \frac{\alpha}{2\pi}\left\{\frac{\alpha_s(M)}{2\pi}\left[\frac{k_q^{(0)}P_{qq}^{(0)}}{2}L_M^2 + k_q^{(1)}L_M\right] + \right. \\ &\qquad \left. + \left(\frac{\alpha_s(M)}{2\pi}\right)^2\left[k_q^{(0)}P_{qq}^{(1)}L_M^2 + \left(k_q^{(2)} - \frac{2k_q^{(1)}P_{qq}^{(1)}}{\beta_0}\right)L_M\right]\right\} \end{aligned} \tag{5.32}$$

and involves in addition to quantities entering the LO formula (5.29), i.e. $k_q^{(0)}, k_q^{(1)}$ and $P_{qq}^{(0)}$ also the $\mathcal{O}(\alpha\alpha_s^2)$ inhomogeneous splitting function $k_q^{(2)}$ and $\mathcal{O}(\alpha_s^2)$ homogeneous splitting function $P_{qq}^{(1)}$. Whereas the latter is known, the former is not, preventing thereby the evaluation of $q_{\rm NLO}$. Furthermore, a complete NLO analysis of $F(Q^2)$ based on (5.10) requires also the knowledge of $\mathcal{O}(\alpha\alpha_s^2)$ photonic coefficient function $C_\gamma^{(2)}$, which is also so far unknown.

6. Phenomenological implications

In this section I will demonstrate numerical importance of the contributions to $F_{\rm LO}(Q^2)$ resulting from inclusion of the quantities omitted in standard LO analysis: the inhomogeneous splitting function $k_q^{(1)}$, photonic coefficient functions $C_\gamma^{(0)}$ and $C_\gamma^{(1)}$ and quark coefficient function $C_q^{(1)}$. Any realistic analysis of experimental data will require taking into account the non-zero value of β_1, as well as extension of the formalism to the singlet channel and incorporation of the contributions of hadronic parts of PDF. Nevertheless, the quantitative impact of some of the effects discussed in this section is so large that we may expect them to affect significantly the complete LO analysis of $F(Q^2)$ as well.

6.1 The effects of $C_\gamma^{(0)}$

Throughout this section I will take for $C_\gamma^{(0)}$ the standard expression (2.16), setting $M^2 = Q^2$. The presence of the lower bound M_0^2 on quark virtuality complicates, however, the situation and I will therefore discuss this point in more detail in subsection 6.6.

6.2 The effects of $C_\gamma^{(1)}$

For illustration of the numerical importance of $C_\gamma^{(1)}$, its contribution to $F_{\rm LO}(Q^2)$ in $\overline{\rm MS}$ + MS scheme is compared in figure 4 to those of $C_\gamma^{(0)}$, the QED formula (5.7), as well as other QCD contributions discussed in this section. The comparison is

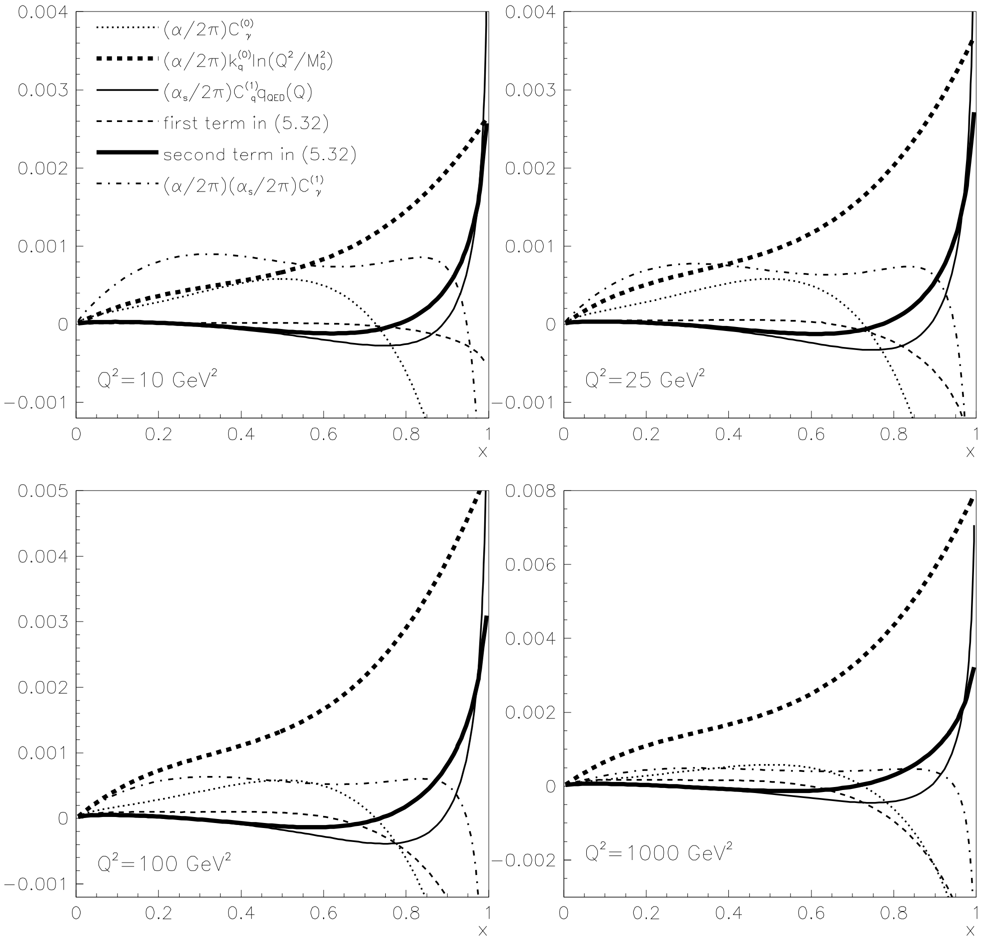

Figure 4: Comparison of individual contributions to $F(x,Q^2)$ as described in the text.

performed for $M^2 = 10, 25, 100$ and $1000\ \mathrm{GeV}^2$. Contrary to the contribution of $C_\gamma^{(0)}$, $C_\gamma^{(1)}$ enters the expression for $F_{\mathrm{LO}}(Q^2)$ multiplied by $\alpha_s(Q)$, and therefore decreases with increasing Q^2 until it becomes negligible with respect to the former as $Q \to \infty$. However, one would have to go to extremely large Q^2, inaccessible in current experiments, for this dominance of $C_\gamma^{(0)}$ to be a good approximation. Figure 4 shows that in the whole currently accessible region of Q^2 and for x up to about 0.85, the term proportional to $C_\gamma^{(1)}$ provides numerically the most important $\mathcal{O}(\alpha\alpha_s)$ corrections.

6.3 The effects of $C_q^{(1)}$

Performing the convolution $C_q^{(1)} \otimes q_{\mathrm{QED}}$ in (5.14), where

$$C_q^{(1)}(x) = C_F\left(\frac{9+5x}{4} - \frac{1+x^2}{1-x}\ln x - \frac{3}{4}\frac{1+x^2}{[1-x]_+} + \right.$$
$$\left. + \left(1+x^2\right)\left[\frac{\ln(1-x)}{1-x}\right]_+ - \left(\frac{9}{2} - \frac{\pi^2}{3}\right)\right) \tag{6.1}$$

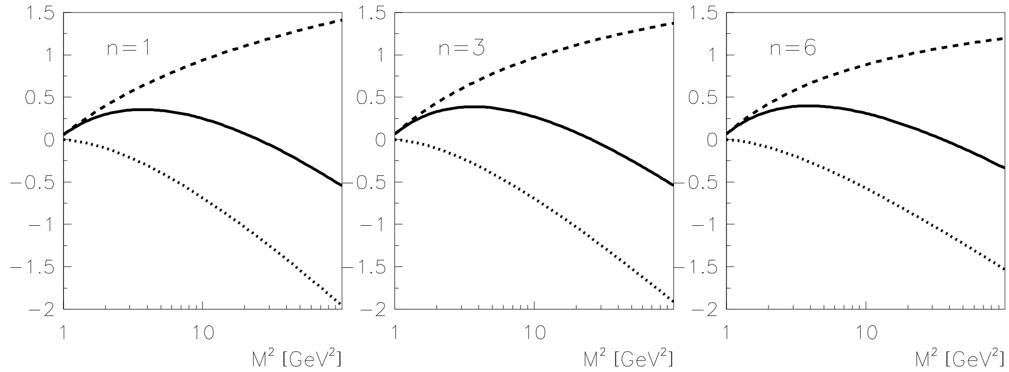

Figure 5: The LO expression (5.29), multiplied by $2\beta_0\pi/\alpha$, as a function of M^2 for three moments $n = 1, 3, 6$, $M_0 = 1$ GeV and $\Lambda = 0.2$ GeV. The solid line shows the full result, the dotted and dashed ones correspond to the first and second term in (5.29), respectively.

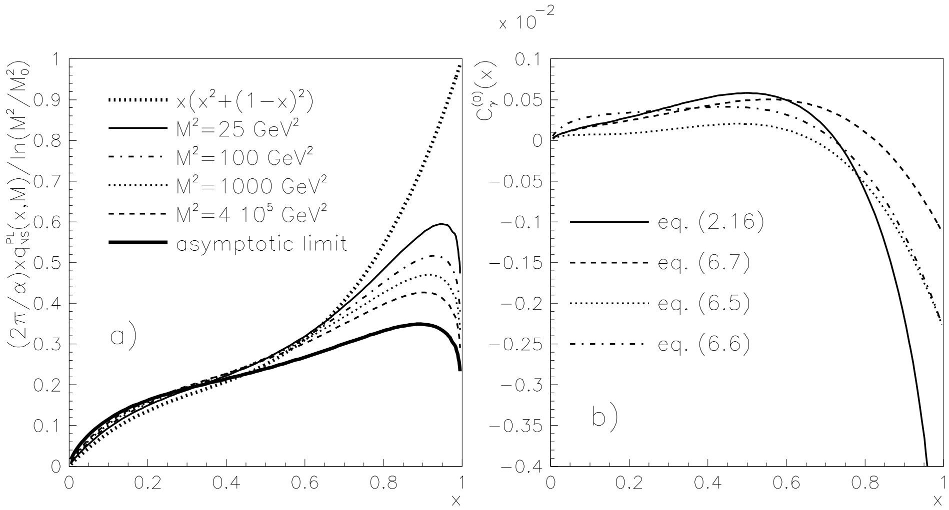

Figure 6: Illustration of the approach to the asymptotic pointlike expression (3.6) (a), and the comparison of three expressions for $C_\gamma^{(0)}(x)$ discussed in the text (b).

is straightforward and leads to thin solid curves in figure 4. This contribution has qualitatively the same shape as that of $k_q^{(1)}$, discussed below, and is vital in the region close to $x = 1$.

6.4 The effects of $k_q^{(1)}$

At asymptotically large values of M^2 the first term on the r.h.s. of (5.29), which is the only one included in the standard approach at the LO, dominates due to the presence of "large log" $\ln(M^2/M_0^2)$ but again one would have to go to very large values of M^2, to see this dominance numerically.[14] This is illustrated in figure 5, where $q_{\rm LO}(M)$, as defined in (5.29), is plotted as a function of M^2 for three moments $n = 1, 3, 6$

[14]For $M_0 \doteq 1$ GeV and currently accessible $M^2 < 200$ GeV^2, $\ln(M^2/M_0^2) \lesssim 5$, hardly a large number.

and together with separate contributions of the first and second terms in (5.29). We see that for $M^2 \leq 100$ GeV^2 the effect of including the latter almost cancels the negative contribution of the term appearing in the conventional analysis. Moreover, converting (5.29) into the x-space by means of inverse Mellin transformation we find (see figure 4) that the negative contribution of the first term in (5.29) to $q_{\mathrm{LO}}(M)$ comes mostly from the region close to $x = 1$. Below $x \doteq 0.75$ the genuine QCD effects described by this part of (5.29) are tiny and smaller than those described by the second term, proportional to the inhomogeneous splitting function $k_q^{(1)}$! The latter gives small negative contribution to $q_{\mathrm{LO}}(x, M)$ up to $x \doteq 0.7$ and large positive one above that value. Figures 4 and 5 indicate that in the whole region of Q^2 and for most of the range of x accessible experimentally, the second term in (5.29) is numerically more important than the one included in standard LO analyses. Only for x close to $x = 1$ are both terms in (5.29) comparable and more or less cancelling each other. The tiny effect of QCD corrections described by the first part of (5.29) reflects the fact that in the region $x \lesssim 0.65$ its contribution to scaling violations of the sum $q_{\mathrm{QED}} + q_{\mathrm{QCD}}$, plotted in figure 6$a$, are very small.

6.5 Summary of QCD contributions at the LO

In figure 7 the sum of contributions to $F_{\mathrm{LO}}(Q^2)$ coming from $C_\gamma^{(1)}, C_q^{(1)}$ and $k_q^{(1)}$ is compared to that given by the first term in (5.29), purely QED formula (5.9), $F_{\mathrm{QED}} + F_{\mathrm{LO}}$ in my (5.14) and standard approaches (4.1). In all calculations $M^2 = Q^2$. The differences between thick solid and thick dash-dotted curves, representing the last two quantities, are large and persists for all experimentally accessible values of Q^2. This is caused by the fact that in the whole kinematical region of Q^2 and for most of the region of x accessible in current experiments, the sum of the three additional contributions included in the LO analysis proposed in this paper, is much larger than the contribution of the term generated by $P_{qq}^{(0)}$. But even for x close to $x = 1$ these additional terms are important as they help cancel part of the negative contribution of $C_\gamma^{(0)}$.

6.6 The large x behaviour

The comparison in figure 7 also shows that the term $\ln(1 - x)$ in (2.16) is much less troubling within the formulation proposed in this paper than within the standard one. The reason is simple: the contributions of additional terms compensate part of the negative contribution of $C_\gamma^{(0)}$ at large x and shift the region where (5.14) turns negative much closer to $x = 1$. In this context it is useful to recall the origins of this term within the parton model derivation of (2.16) in which the parallel singularity associated with the splitting $\gamma \to q\overline{q}$ is regularized by means of quark masses and the troubling term $\ln(1 - x)$ comes from the lower bound on the quark virtuality τ

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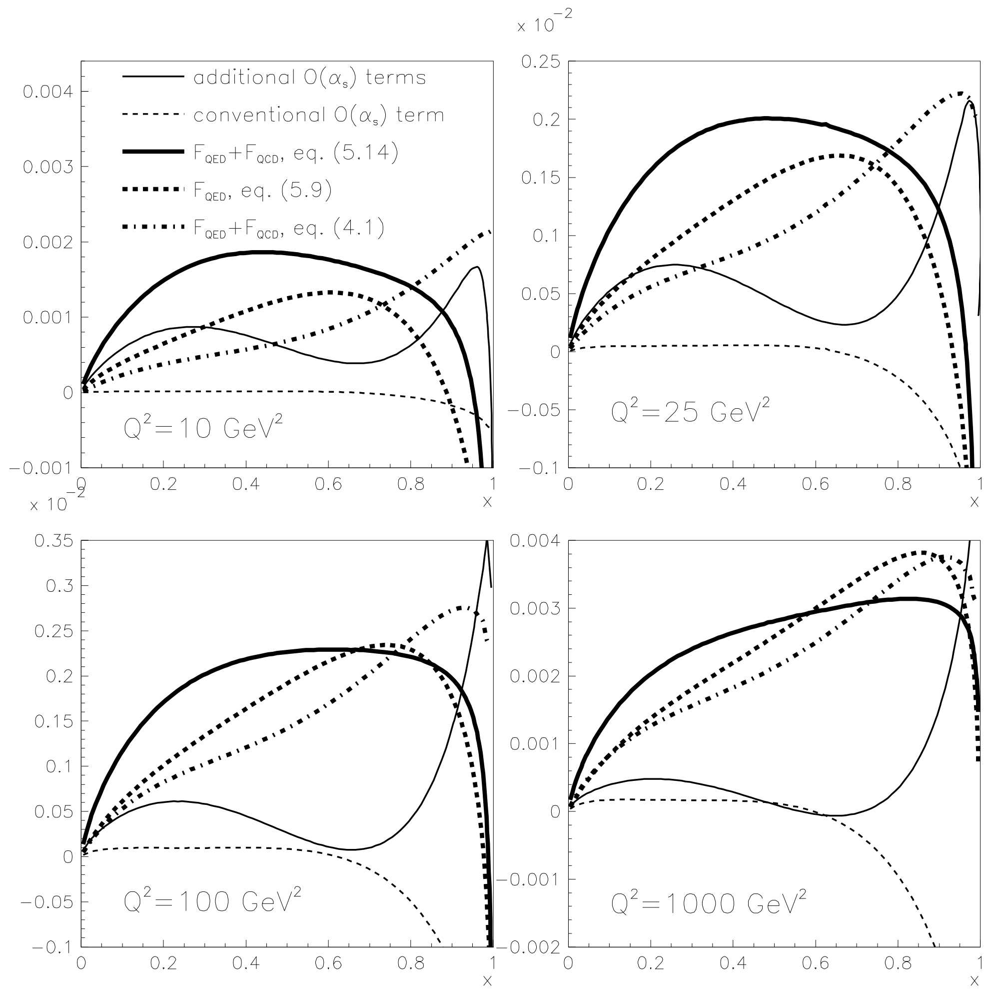

Figure 7: The sum of $\mathcal{O}(\alpha_s)$ contributions to $F_{\mathrm{LO}}(x, Q^2)$ included in this analysis, but omitted in the conventional one (thin solid curves), compared to those of $P_{qq}^{(0)}$ (thin dashed curves), QED formula (5.9), as well as $F_{\mathrm{QED}} + F_{\mathrm{LO}}$ in my (5.14) and standard (4.1) approaches.

(see figure 1) in collinear kinematics

$$\tau_{\min} \equiv \frac{m_q^2}{1-x} \leq \tau \leq \tau_{\max} \equiv \frac{Q^2}{x} \,. \tag{6.2}$$

Performing the integration over τ in the limits $\tau_{\min} \leq \tau \leq M^2$, the primary QED splitting $\gamma \to q\overline{q}$ yields the expression proportional to

$$f(x)\ln\left(\frac{M^2}{\tau_{\min}}\right) = f(x)\ln\left(\frac{M^2(1-x)}{m_q^2}\right) = f(x)\ln\left(\frac{M^2}{m_q^2}\right) + f(x)\ln(1-x)\,, \tag{6.3}$$

where $f(x) = x^2 + (1-x)^2$. In the conventional approach the term $f(x)\ln(M^2/m_q^2)$ is included in the quark distribution function $q(x, M)$, whereas $f(x)\ln(1-x)$ goes into $C_\gamma^{(0)}$. The complete expression for nonlogarithmic term of (2.16) includes, beside the term $f(x)\ln(1-x)$, also terms coming from the upper bound in (6.2) as well as from lower bound in integrals over the terms m_q^2/τ^2, or $1/s$. In these latter cases the dependence on m_q enters $F(x, Q^2)$ through the multiplicative factor

$$c \equiv 1 - \frac{\tau_{\min}}{M^2} = 1 - \frac{m_q^2}{M^2(1-x)}\,, \tag{6.4}$$

where we can set $m_q = 0$. Moreover, as the ratio $\tau_{\min}/M^2$ is formally of power correction type, we can neglect it even for $m_q > 0$. The term $\ln(1-x)$ in $C_\gamma^{(0)}$ causes problems in phenomenological parameterizations primarily because it appears there decoupled from the value of the quark mass m_q with which it originally entered the expression (6.2) and thus persists there even in the limit $m_q \to 0$, or when the quark mass is replaced by the initial scale M_0.

In the presence of the lower cutt-off M_0^2, $\tau_{\min}$ in (6.2) should perhaps be replaced with $\tau_0 \equiv \max(m_q^2/(1-x), M_0^2)$. Sending $m_q \to 0$, but even for all realistic values of light quark masses and accessible x, we get $\tau_0 = M_0^2$. This implies the replacement $\ln((1-x)/x) \to \ln(1/x)$ in $C_\gamma^{(0)}$, thereby removing most of the practical problems with the term $\ln(1-x)$ in (2.16).

Beside $\ln(1-x)$ there is another term in $C_\gamma^{(0)}$ that comes from non-zero quark mass, namely the last "+1" in the non-logarithmic part $8x(1-x) - 1 = 8x(1-x) - 2 + 1$. Discarding also this remaining trace of m_q we get

$$C_\gamma^{(0)}(x,1) = \left[x^2 + (1-x)^2\right]\ln\frac{1}{x} + 8x(1-x) - 2\,. \tag{6.5}$$

This expression is close to, but not identical with the QED formula for the constant part of $F(x, Q^2)$ obtained for massless quarks coupled to photon with non-zero virtuality $P^2 > 0$ [21]

$$C_\gamma^{(0)}(x,1) = \left[x^2 + (1-x)^2\right]\ln\frac{1}{x^2} + 8x(1-x) - 2\,. \tag{6.6}$$

The additional $1/x$ in the logarithm $\ln(1/x^2)$ with respect to (6.5) reflects the fact that for $m_q = 0$ and $P^2 > 0$ the lower bound $\tau_{\min} = xP^2$. The expression (6.5) for $C_\gamma^{(0)}$ is also close to that used in [13]

$$C_\gamma^{(0)}(x,1) = \left[x^2 + (1-x)^2\right]\ln\frac{1}{x} + 6x(1-x) - 1\,. \tag{6.7}$$

In figure 6*b* all three expressions (6.5)–(6.7) are compared to the standard one of eq. (2.16).

7. Conclusions

I have proposed a reformulation of QCD analysis of F_2^γ, which separates genuine QCD effects from those due to QED and satisfies basic requirement of factorization scale invariance at finite orders of perturbative QCD. This reformulation differs substantially from the conventional approach.

At the LO it requires the inclusion of four terms, proportional to $C_q^{(1)}, k_q^{(1)}, C_\gamma^{(0)}$ and $C_\gamma^{(1)}$, all of which are known. Whereas in the standard approach the first three of them are part of the NLO approximation, the one proportional to $C_\gamma^{(1)}$, enters the standard formulation first at the NNLO. Detailed numerical analysis of the contributions of these terms shows that in most of the kinematical region currently accessible experimentally their sum is more important than the $\mathcal{O}(\alpha_s)$ contribution included in the standard LO expression for F_2^γ. It turns out that the part of $C_\gamma^{(0)}$ proportional to $\ln(1-x)$, which causes serious problems in the standard formulation in the region $x \to 1$, is much less troubling in the proposed reformulation. All quantitative considerations were carried out for the pointlike part of F_2^γ in the non-singlet channel and assuming $\beta_1 = 0$.

Contrary to the conventional approach, the NLO analysis of F_2^γ proposed in this paper requires the knowledge of two so far uncalculated quantities and is thus currently impossible to perform.

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